Parametrised h-Cobordism Theorem

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The goal is to discuss and explain the linking factor between geometric topology and K theory, that is the (stable) parametrised h-cobordism thoerem [WJR, Thm 0.1]. This theorem declares two spaces to be homotopy equivalent, and so all I really want to do is explain *what* those spaces are, so that at least the statement of the theorem is understandable. The references we have looked at are [Hat78], [Wal82], [Wal85] and [Kup].

1 Loosely Speaking

Consider M a $d \geq 5$ closed connected manifold and $\pi = \pi_1(M)$. A h-cobordism on / of M is a $d+1$ dimensional manifold W such that M is one of two boundary components of W and the inclusion of both boundary components (individually) into W are homotopy equivalences. Smales h-cobordism theorem says that if $\pi = 1$ then a h-cobordism is always diffeomorphic to the cylinder $M \times I$. The s-cobordism theorem of Mazur, Stallings and others, says that there is an isomorphism of sets

$$\tau(-, M) : \{\text{h-cobordisms of M}\}/(\text{iso}) \xrightarrow{\sim} \mathrm{Wh}_1(\pi) \quad := \quad K_1(\mathbb{Z}[\pi])/(\pm\pi)$$

Moreover if $W_1 : M \to N$ is a h-cobordism and $W_2 : N \to P$ is another then this bijection acts like a group homomorphisms, that is

$$\tau(W_1 \cup_N W_2, M) = \tau(W_1, M) + \tau(W_2, N)$$

noting that because $M \simeq N$ their fundamental groups are the same and so $\tau(-, M), \tau(-, N) \in \mathrm{Wh}_1(\pi)$.

The goal of the parametrised h-cobordism theorem is to replace both sides of this isomorphism with a *space* and replace the isomorphism of sets with a weak homotopy equivalence. That is there should be spaces $H(M)$ and $\mathrm{Wh}^{\mathrm{Diff}}(M)$ such that the s-cobordism theorem reduces to the statement

$$\pi_0 H(M) \cong \pi_1 \mathrm{Wh}^{\mathrm{Diff}}(M)$$

and that further we have a weak equivalence

$$H(M) \simeq \Omega \mathrm{Wh}^{\mathrm{Diff}}(M).$$

Following the philosophy of classifying spaces, which says at least for a group that we have $\pi_0 BG \cong G$ is its reasonable to think that we should define $H(M)$ as a classifying space for bundles of h-cobordisms. That is a homotopy class of maps $X \to H(M)$ should define a bundle over X whose fibers are h-cobordisms of M. So since we already have the classical s-cobordism theorem we can make some progress constructing such a space. Usually if you want to classify bundles whose fibers are given by W then you look at the space $B\mathrm{Diff}(W)$. In this case we want bundles of h-cobordisms, of which there is more than one diffeomorphism type of, but at least if we wanted a bundle of h-cobordisms of type $[W] \in \mathrm{Wh}_1(\pi)$ then we would look at $B\mathrm{Diff}(W; M)$, requiring the diffeomorphism to be relative to M, to preserve that it is a cobordism of M. Then it is clear that we can just collect all these things together so

$$H(M) \simeq \coprod_{[W]} B\mathrm{Diff}(W; M)$$

or if we denote $H(M)_\tau$ the path component of $H(M)$ which contains $[W] = \tau$ we get that

$$H(M) = \coprod_{\tau} H(M)_\tau.$$

Lets look at $H(M)_0$. This is the path component of the trivial h-cobordism, $M \times I$, the diffeomorphisms of $M \times I$ that are relative to M are exactly the group of concordances however so we have that $H(M)_0 = B\mathscr{C}(M)$! On the other hand we claim that

Lemma. *There is a homotopy equivalence $H(M)_\tau \simeq H(M)_0$ for all τ.*

Proof. Consider two h-cobordisms, $W_1 : M \to M_\tau, W_2 : M_\tau \to M$ who have Whitehead torsion τ and $-\tau$ respectively Not immediate that this is possible, namely why must the class of h-cobordisms that have opposite whitehead torsion swap the to and from, is it true that $\tau(M, N) = -\tau(N, M)$? Then by the sum formula for Whitehead tosion we get that

$$W_1 \cup_{M_\tau} W_2 \cong M \times I$$

This defines a map

$$H(M)_0 \to B\mathrm{Diff}(W_1 \cup_{M_\tau} W_2) \to H(M)_\tau$$

which corresponds on the level of diffeomorphisms to restricting to one half of the cylinder, which it is claimed is a homtopy equivalence. The inverse map that they provide is given by glying $W_1 \cup_M W_2$ but this doesnt make sense to me because then its not going to have M as a boundary component and so it shouldnt land where they claim, namely in $H(M)$, it would be in $H(M_\tau)$? Kupers has the same argument.

Thus we can replace all the $H(M)_\tau$ with $H(M)_0$ and get that

$$H(M) \simeq \coprod_{\tau} H(M)_0 \simeq \mathrm{Wh}_1(\pi) \times B\mathscr{C}(M).$$

Unfortunately we have not been able to produce such a friendly story for the construction of $\mathrm{Wh}^{\mathrm{Diff}}$, thus we shall move on to more rigid considerations. Let us be a bit more clear also, this program has not yet been carried out, what has been shown only is that if one *stabilises* this space then we obtain the weak equivalence.

2 Whitehead Spectrum

The fundamental property of the Whitehead spectrum is that it sits in a fibration

$$\Sigma^\infty X_+ \to A(X) \to \mathrm{Wh}^{\mathrm{Diff}}(X)$$

to get the definition then there are two approaches. The traditional one I guess is to specify the Whitehead space using some horrible simplicial models and then construct a map from the A theory and show that the fiber is $\Sigma^\infty X_+$. The other way is to construct a map from $\Sigma^\infty X_+$ to $A(X)$ and define the Whitehead space as its cofiber. Neither are very nice and it is not obvious from either why its homotopy groups should have anything to do with either of K_1 or $H(M)$.

2.1 Functorial Definition

The smoothest definition we have found is a purely abstract one, this is the one found in [WJR]. It gives you lots of universal properties to play with, but at least for me fails to make clear *why* this thing should relate to K theory or h-cobordisms.

One starts with an arbitrary functor $F : \text{Spaces} \to \text{Spectra}$ that preserves weak equivalences, that is homotopy invariant. Then [WW95] provide an approximation of this functor by what they call a strongly excisive homotopy invariant functor, basically something more like a homology. So take $\mathrm{simp}(X)$ to be the cateogory of simplicies in a space X, that is its objects are maps

$$\Delta^n \to X$$

and its morphisms are maps $\Delta^m \to \Delta^m$ that are over X and moreover come from monotone injections $[m] \to [n]$ (that is morphisms in the simplex category). There is a functor then

$$F_X : \mathrm{simp}(X) \to \text{Spectra}$$

induced by F, simply by $F_X(g : \Delta^n \to X) \ := \ F(\Delta^n)$, note that because F is homotopy invariant and $\Delta^n \simeq *$ it is clear that *up to homotopy* this is a constant functor, its image is always in the same homotopy class of spectra. They then define a new functor that is given pointwise by the homotopy colimits of these F_X

$$F^{\%}(X) \ := \ \mathrm{hocolim} F_X$$

This defines a new functor that is "strongly excisive" and because it is constant up to homotopy, and the general theory of homotopy colimits for constant functors, we know that it is weakly equivalent to the functor

$$X \mapsto X_+ \wedge F(*).$$

Finally there is the natural transformation that sends $F^{\%} \to F$. There is a natural map

$$F^{\%}(X) \to F(X)$$

which is induced under the hocolim by the natural transformation $F_X \to F(X)$ where $F(X)$ is the constant functor, which comes from assembling all the maps

$$F(g : \Delta^n \to X) = \alpha_g : F_X(g) \to F(X)(g) = F(X).$$

Thus we have a map induced basically by hocolims

$$X_+ \wedge F(*) \xrightarrow{\alpha} F(X)$$

which [WW95] call the assembly map.

Applying this to the case of A theory we get an assembly map

$$X_+ \wedge A(*) \to A(X)$$

which we can precompose with the unit map of (some type of ring) spectrum $\eta : \mathbb{S} \to A(*)$ thus producing a map

$$\Sigma^\infty X_+ = X_+ \wedge \mathbb{S} \xrightarrow{\mathrm{id}\wedge\eta} X_+ \wedge A(*) \xrightarrow{\alpha} A(X)$$

taking the cofiber, the homotopy pushout, of this diagram then gives $\mathrm{Wh}^{\mathrm{Diff}}$.

Remark. This setup is nice to see the difference between the PL and smooth case. In the PL case we have that $\mathrm{Wh}^{\mathrm{PL}}$ is the cofiber of the diagram

$$X_+ \wedge A(*) \xrightarrow{\alpha} A(X)$$

and so the only difference is a precomposition with the unit map from the sphere spectrum.

Remark. (ho-limits) The first thing to note is that pushout and pullback squares coincide in the category of spectra. Next the category of spectra is pointed, with the terminal object we will denote $*$. In such a cateogy one can describe fiber sequences, or dually cofiber sequences, as (homotopy) pullbacks or (homotopy) pushouts. Because they coincide in spectra we can do this in one diagram:

$$\begin{array}{ccc} \mathrm{fib}(g) & \xrightarrow{f} & X \\ \downarrow & & \downarrow{\scriptstyle g} \\ * & \longrightarrow & \mathrm{cofib}(f) \end{array}$$

2.2 The PL case

The case of Wh^{PL} can give some helpful insights into what the sort of idea behind the Whitehead space is *supposed* to be, although it is not directly helpful. This is a sort of warm up to the manifold models which are a bit messier but ultimately similar in spirit.

In [Wal85] we are given the following definition of the Whitehead space. First similar to A theory we define for a given space X (a simplicial set actually) the category $C^h_f(X)$ of finite contractable cofibrant spaces over X (compare finite retractive spaces). Then the Whitehad space is

$$\mathrm{Wh}^{\mathrm{PL}}(X) \;\; := \;\; sN_\bullet C^h_f(X)$$

given by the subcategory of *simple maps* in the simplicial category given by taking *the nerve* of $C^h_f(X)$.

Waldhausen (Thm 3.1.7) then shows that this is weakly equivalent to several other things but in particular

$$\mathrm{Wh}^{\mathrm{PL}} \simeq sS_\bullet R^h_f(X^{\Delta^\bullet})$$

There is as far as I can tell no straightforward way to compare the right hand side to $A(X)$, that is no obvious maps on the level of Waldhausen categories, but it does show the relation of the Whitehead space to $A(X)$ a bit more clearly. Just to clarify this is the simple maps inside of the simplicial category produced by the $S_\bullet$ construction, applied to the finite retractive and contractable spaces over $X^{\Delta^\bullet} = \mathrm{Hom}_{sSet}(\Delta^\bullet, X)$.

Remark. There is some resemblance here with the assembly construction. You are looking at the space of simplicies in X, I dont know.

Remark. This definition still doesnt make it obvious how it would be connected to K theory or h-cobordism, although it is at least parsable.

2.3 The Q Construction

[Kup] gives yet another description of the map $\Sigma^\infty X_+ \to A(X)$ that may or may not be helpful.

2.4 Relation to K_1

[Kup] gives an explanation, it is not straightforward and involves another spectral sequence. I consider this unfinished business to figure out the intuitive relation.

3 Manifold Models

Waldhausen in [Wal82] gives some manifold models of both the Whitehead space and $H(M)$. As a summary he defines a simplicial set of partitions of $M \times I^k$, then certain partitions will corespond to h-cobordisms. Finally using constructions of Segal and stabilising one can get the Whitehead space. Add the images of Waldhausen, they are indispensable.

3.1 Partitions

Let X be a compact manifold with boundary. Then a partition of $X \times I$ is a submanifold M that satisfies the following properties

- The "frontier" defined as the intersection between M and the closure of its compliment, and denoted $F = M \cap \overline{M^c}$, should be disjoint from the top and bottom edges, $X \times \{0, 1\}$.
- F should be standard near the boundary $\partial X \times I$, that is there should be a neigbourhood such that intersecting with F is the same as intersecting with $X \times \{t\}$ for some t. "F comes in perpendicular to the boundary".

Note that there is nothing preventing the submanifold from not being simply connected, etc. These partitions form the 0-simplicies of a simplicial set $P(X)$ which by definition has k simplicies given by locally trivial families of partitions parametrised by Δ^k. That is a k-simplex is a locally trivial map

$$P_0(X) \to \Delta^k$$

Doesnt this mean that $P_0(X)$ which is just some subset of the submanifolds of X has a smooth structure? What is it? Because Wald wants this locally trivial map to be smooth...

The h-cobordism space $H(X)$ is then defined to be the (simplicial) subset of $P(X)$ such that M is a h-cobordims (necissarily bewteen F and X embedded as $X \times \{0\}$). So far so good, $H(X)$ is (at least on zero simplicies) just all h-cobordisms between X and a frontier that can be arbitrary. From here on things begin to get more technical.

First notice that $P(X)$ forms a simplicial poset because the M's may or may not include into one another, because of the functor Poset $\to$ Cat we can therefore consider $P(X)$ as a simplicial category. This simplicial category has a simplicial subcategory $hP(X)$ which is given by

- looking only at the morphisms $M \to M'$ that are homotopy equivalences
- *and* that the two frontiers, F and F', when included into $X \times I$ should cobound a h-cobordism "in the middle". Precisely we require that inclusions of the frontiers into $M' - (M - F)$ should be homotopy equivalences, and if we consider "general position" that is up to homotopy we can take the inclusions to be disjoint, then we get that they cobound an h-cobordism.

Pointwise, or as simplicial sets, however there is no difference between $P(X)$ and $hP(X)$, only as simplicial categories. Loosely as sets they are the collections of frontiers, as categories the latter is the collection of frontiers, *up to h-cobordism.*

Next we have the simplicial category $hP_k^m(X)$ which is the connected compoent of $hP(X)$ which contains the submanifold $X \times [0, 1/2] \cup (k$ trivial $m-$ handles). The superscript is the dimension of the handles, the subscript is the number of them. $P_k^m(X)$ is the underlying simplicial set for this simplicial category. There is moreover a map $hP_k^m(X) \to hP_{k+1}^m(X)$ "given by adding a k handle in some standard way". This map will be used in stabilisation later. It should be noted that there is an identification $H(X) = P_0^m(X)$ This is intuitive almost, but I would like to think about it more.

3.2 Stabilisation and Composition

The next step is to define the stabilisations of these spaces, this involves some technical points, especially because the partitions are required to be standard near the boundary. Waldhausen defines, for some choices of submanifolds of X some spaces that are easier to stabilise, $\underline{P}(X)$, which are at the end of the day homotopy equivalnet to $P(X)$. We will skip over this technical point and conflate the two. The stabilisation map is then given by (loosely)

$$P(X) \to P(X \times I)$$

By more or less sending a submanifold M to $M \times I$. This in essence is describing a natural transformation between the functors P and $P(- \times I)$, but only up to coherent homotopy.

Now we can take the limit over these stabilisation maps to get the stable space of partitions

$$\mathcal{P}(X) \quad := \quad \lim_n P(X \times I^n)$$

Which should be considered as at least a simplicial set (possibly a simplicial category), although Waldhausen refers to it as a space, which for him means a topological space, I assume that limits commute with geometric realisation...?. The stable space of H cobordisms,

$$\mathcal{H}(X) \quad := \quad \lim_n H(X \times I^n)$$

The stable space of $hP(X)$

$$h\mathcal{P}(X) \quad := \quad \lim_n hP(X \times I^n)$$

and the stable space of m handled partitions

$$\mathcal{P}_s^m(X) \quad := \quad \lim_{n,k} P_k^m(X \times I^n), \text{ or } \mathcal{P}^m(X) \quad := \quad \lim_n \left(\coprod_k P_k^m(X \times I^n) \right)$$

Then the claim is that these simplicial sets, or the spaces associated to them, are partial monoids.

A partial monoid is what is sounds like, but for completeness we record the definition from [Seg73]; A space M with some subspace $M_2 \subseteq M \times M$ and a map $\cdot : M_2 \to M$ is a partial monoid if

- There is a unit $1 \in M$ such that $1 \cdot m = m \cdot 1$ for all $m \in M$, in particular both sides of the equation are required to be defined
- And when one side of the following equation is defined so is the other and they are equal

$$m \cdot (m' \cdot m'') = (m \cdot m') \cdot m''$$

Segal associates to such a partial monoid a simplicial space assigning

$$M_\bullet : [n] \mapsto \text{ composable } n \text{ tuples in } M$$

Now $P(X)$ has a composition given by taking the union of M and M' inside $X \times I$. Note however that to make this union a manifold we require that the two submanifolds have "disjoint support", a technical condition that is not always possible to satisfy, hence making $P(X)$ only a partial monoid. In the stable limit however there is always room to move the two manifolds into a sort of general position ensuring that they, up to homotopy, always have disjoint support. This gives the stable space much nicer properties (its probably in infinite loop space but I havent read segal closely).

The simplicial space associated to this partial monoid Waldhausen denotes $N_\Gamma \mathcal{P}(X)$, or one can replace $\mathcal{P}$ with the other stabilised spcaes such as $h\mathcal{P}, \mathcal{H}$ etc. Finally then we have the definition that he gives

$$\mathrm{Wh}^{\mathrm{Diff}}(X) \quad := \quad N_\Gamma \mathcal{H}(X).$$

3.3 Relations to A Theory

The rest of his manifold approach paper is deducing useful maps between all these spaces and in particular the map that exhibits the Whitehead space in the fibration above. We will discuss these loosely here. Recall that for a fibration

$$F \to E \to B$$

we have up to homotopy a sequence

$$\Omega B \to F \to E \to B$$

given by the homotopy fiber / Puppe sequence construction [Hat02, §4.6] where the first two and last two seperatly form fibrations up to homotopy. Now in [Wal82] the first theorem he states loosly as a fibration

$$H(X) \to P_k^m(X) \to hP_k^m(X)$$

and a weak equivalence between $\Omega h P_k^m(X)$ and $A(X)$, hence we have a sequence

$$\Omega A(X) \to H(X) \to P_k^m(X) \to A(X)$$

In making this theorem precise the first map is to become the cofiber map $A(X) \to \mathrm{Wh}^{\mathrm{Diff}}(X)$.

So what is the precise statement. In essence it is taking each of these things to their appropriate stabilisations and applying N_Γ to them.

Lemma ([Wal82], Prop 5.1)**.** *There is a homotopy pushout / pullback square in a range (based on m) given by*

$$\begin{array}{ccc} \mathcal{H}(X) & \longrightarrow & \mathcal{P}_s^m(X) \\ \downarrow & & \downarrow \\ h\mathcal{H}(X) & \longrightarrow & h\mathcal{P}_s^m(X) \end{array}$$

Applying the "plus construction" to the RHS of this diagram (notice the missing s subscript) and taking limits gets

Lemma ([Wal82], Prop 5.1)**.** *There is a homotopy pushout / pullback square*

$$\begin{array}{ccc} \mathrm{Wh}^{\mathrm{Diff}}(X) = N_\Gamma \mathcal{H}(X) & \longrightarrow & \lim_m N_\Gamma\big(\mathcal{P}^m(X)\big) \\ \downarrow & & \downarrow \\ N_\Gamma h\mathcal{H}(X) & \longrightarrow & \lim_m N_\Gamma\big(h\mathcal{P}^m(X)\big) \end{array}$$

Finally we have the weak equivalence

Lemma ([Wal82], Prop 5.4)**.** *There is a highly connected natural transformation*

$$\Omega N_\Gamma h\mathcal{P}^m(X) \to A(X)$$

so in particular a weak equivalence

$$A(X) \simeq \lim_m \Omega N_\Gamma h\mathcal{P}^m(X)$$

Loop has an adjoint so probably commutes with these limits, moreover since these spaces are all infinite loop spaces they are CW complexes and therefore a weak equivalence can be upgraded to a homotopy equivalence, thus the map from $\Omega N_\Gamma h\mathcal{P}^m(X) \to A(X)$ can be seen to have an inverse $A(X) \to \Omega N_\Gamma h\mathcal{P}^m(X)$ which we compose along with the Puppe map to get

$$A(X) \xrightarrow{\sim} \Omega \lim_m N_\Gamma h\mathcal{P}^m(X) \to N_\Gamma \mathcal{H}(X) = \mathrm{Wh}^{\mathrm{Diff}}(X).$$

3.4 Relation to G/O

Finally in [Wal82] there are some maps relating A and $\mathrm{Wh}^{\mathrm{Diff}}$ to BG and G/O which are nice to discuss. The goal is to construct a morphism of diagrams, such that we get an induced map on the fibers:

$$\begin{array}{ccccc} G/O & \dashrightarrow & BO & \longrightarrow & BG \\ \vdots & & & \Downarrow & \\ \Omega\mathrm{Wh}^{\mathrm{Diff}}(*) & \dashrightarrow & T^{\mathrm{Diff}} & \longrightarrow & BG \end{array}$$

The connective tissue between the previous sections and this map is the space T^{Diff}, this is the limit over some other space.

$$T_{m,n} = \text{set of “tube of type (m,n)”}$$

Where a tube of type (m,n) is an m handle attached to $\mathbb{R}^{m+n} \times (-\infty, 0]$ in an unknotted fashion. Then $T^{m,n}$ is the simplicial set with k simplicies given by (m,n) tubes over Δ^k. Finally

$$T^{\mathrm{Diff}} \quad := \quad \lim_{m,n} T^{m,n}.$$

Now Waldhausen claims, and gives no explanation for the following, I havent thought about it, maybe its obvious:

Lemma. *There is a weak equivalence* $\lim_m \lim_n hP_1^m(D^{m+n}) \simeq BG$*.*

Lemma. *There is a weak equivalence* $\lim_m \lim_n P_1^m(D^{m+n}) \simeq T^{\mathrm{Diff}}$*.*

Lemma. *There is a weak equivalence* $\mathcal{H}(D^{m+n}) \simeq \Omega\mathrm{Wh}^{\mathrm{Diff}}(*)$*.*

Then a similar fibration to theorem one gives

$$\mathcal{H}(D^{m+n}) \to \lim_m \lim_n P_1^m(D^{m+n}) \to \lim_m \lim_n hP_1^m(D^{m+n})$$

and hence

$$\Omega\mathrm{Wh}^{\mathrm{Diff}}(*) \to T^{\mathrm{Diff}} \to BG$$

Finally we need the morphism between the two diagrams which is given by

Lemma ([Wal82], Prop 3.2)**.** *There is a map* $BO \to T^{\mathrm{Diff}}$ *whose composition with* $T^{\mathrm{Diff}} \to BO$ *is the J-homomorphism.*

Proof. This map Waldhausen describes as "an inclusion of the subspace of rigid tubes". He gives a more explicit construction, but its still not entirely clear to me.

In other words the following diagram commutes

$$\begin{array}{ccc} BO & \xrightarrow{J} & BG \\ \downarrow & & \| \\ T^{\mathrm{Diff}} & \longrightarrow & BG \end{array}$$

and so we get an induced map between the homotopy fibers (pullbacks) of these two diagrams, $G/O \to \Omega \mathrm{Wh}^{\mathrm{Diff}}(*)$.

[Rog02, Thm 7.5] calls this map the "Hatcher-Waldhausen" map and shows that it is precisely 9-connected on two primary homotopy groups, that is on $\pi_*, * \leq 8$. There is a rational equivalence between G/O and BO (coming from the LES and finiteness of the homotopy groups of spheres) and it is asked whether the map to $\Omega \mathrm{Wh}^{\mathrm{Diff}}(*)$ is also an equivalence rationally, it is now known that these two groups are rationally isomorphic (however I dont know if its this map that provides such an iso).

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